Appendix C

Suspension mode control

C.1 Position control system transfer function

This section details the derivation of the Laplace transfer function of the suspension position control system illustrated in Figure 4.4. The Laplace transform of the control law for the position controller is given by:

$$F_{demand}(s) = -\left(k_{pos} + \frac{k_{pos}}{sT_{err}}\right)\left(\frac{s^2 Z(s)}{(s+\omega_{int})^2} - Z_{ref}(s)\right) - k_{vel}\frac{s^2 Z(s)}{(s+\omega_{int})} - k_{acc}s^2 Z(s) \qquad \textbf{C.1}$$

where Z is the position, Z_{ref} is the position reference, and the feedback gains are denoted by: k_{pos} - position error (stiffness), T_{err} - error integral (time constant), k_{vel} - velocity (damping), k_{acc} - acceleration (virtual mass), and ω_{int} is the low frequency cutoff point for the state integrators. This equation is rearranged for later convenience, giving:

$$\begin{aligned} F_{demand}(s) = &-\frac{Z(s)}{(s+\omega_{int})^2}\left[s\frac{k_{pos}}{T_{err}} + s^2 k_{pos} + s^2(s+\omega_{int})k_{vel} + s^2(s+\omega_{int})^2 k_{acc}\right] \\ &+ \frac{Z_{ref}(s)}{s}\left[\frac{k_{pos}}{T_{err}} + s\,k_{pos}\right] \end{aligned} \qquad \textbf{C.2}$$

The acceleration of the suspended load due to the control force demand and a disturbance force is given by:

$$s^2 Z(s) = \frac{F_{demand}(s)}{m\left(1+sT_{force}\right)} + \frac{F_{disturb}(s)}{m} \qquad \textbf{C.3}$$

where $F_{disturb}$ is the disturbance force, T_{force} is the force actuation time constant, and m is the suspended mass. This is also rearranged for convenience, giving:

$$F_{demand}(s) = s^2\left(1+sT_{force}\right)m\ Z(s) - \left(1+sT_{force}\right) F_{disturb}(s) \qquad \textbf{C.4}$$

The Laplace transform of the closed-loop position/disturbance force transfer function can now be determined by equating the control force demand of Equation C.2 and Equation C.4, this gives:

$$\frac{Z(s)}{\left(s+\omega_{int}\right)^2}\left[s\frac{k_{pos}}{T_{err}} + s^2k_{pos} + s^2\left(s+\omega_{int}\right)k_{vel} + s^2\left(s+\omega_{int}\right)^2k_{acc} + s^2\left(1+sT_{force}\right)\left(s+\omega_{int}\right)^2 m\right]$$
$$= \frac{Z_{ref}(s)}{s}\left[\frac{k_{pos}}{T_{err}} + s\,k_{pos}\right] + F_{disturb}(s)\ \left(1+sT_{force}\right) \qquad \textbf{C.5}$$

As an intermediate step, it is useful to expand the terms which comprise the characteristic polynomial of the closed-loop transfer function. The characteristic polynomial is thus given by:

$$CharPoly(s) = s\frac{k_{pos}}{T_{err}} + s^2k_{pos} + s^3k_{vel} + s^2\omega_{int}k_{vel} + s^4k_{acc} + 2s^3\omega_{int}k_{acc} + s^2\omega_{int}^2k_{acc}$$
$$+ s^4m + 2s^3\omega_{int}m + s^2\omega_{int}^2m + s^5mT_{force} + 2s^4\omega_{int}mT_{force} + s^3\omega_{int}^2mT_{force} \qquad \textbf{C.6}$$

Equation C.5 can now be rearranged with the help of Equation C.6 to obtain the closed-loop transfer function of the position control system in terms of the position reference input and the disturbance force input. This yields:

$$\frac{Z(s)}{Z_{ref}(s)} = \frac{k_{pos}\ \left(s+1/T_{err}\right)\ \left(s+\omega_{int}\right)^2}{s^6\left(mT_{force}\right) + s^5\left(k_{acc}+m+2\omega_{int}mT_{force}\right) + s^4\left(k_{vel}+2\omega_{int}(k_{acc}+m)+\omega_{int}^2mT_{force}\right) + s^3\left(k_{pos}+\omega_{int}^2(k_{acc}+m)+\omega_{int}k_{vel}\right) + s^2\left(k_{pos}/T_{err}\right)} \qquad \textbf{C.7}$$

$$\frac{Z(s)}{F_{disturb}(s)} = \frac{\left(1+sT_{force}\right)\left(s+\omega_{int}\right)^2}{s^5\left(mT_{force}\right) + s^4\left(k_{acc}+m+2\omega_{int}mT_{force}\right) + s^3\left(k_{vel}+2\omega_{int}(k_{acc}+m)+\omega_{int}^2mT_{force}\right) + s^2\left(k_{pos}+\omega_{int}^2(k_{acc}+m)+\omega_{int}k_{vel}\right) + s\left(k_{pos}/T_{err}\right)} \qquad \textbf{C.8}$$

C.2 ACSL model of electromagnet suspension control system

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Program Maglev
"---- Single Electromagnet Suspension System Simulation"

"---- Features: - Nonlinear electromagnet model"
"----           - Nonlinear force controller"
"----           - Absolute position controller (P+I+D+DD)"
"----           - Guideway following filter (2 pole)"

"---- Author: Neil McLagan"
"---- Last modified: 4-2-91"

"---- All variables are in elementary units (eg. secs, metres, newtons)"
"---- unless the variable name suggests otherwise, eg. gap.mm"

Initial
    "---- Electromagnet characteristics"
    Constant  Rcoil = 0.79
    Table  Lcoil, 1, 3 / 0.001, 0.003, 0.005,   0.088, 0.067, 0.059 /
    Table  Teddy, 1, 3 / 0.001, 0.003, 0.005,   0.005, 0.003, 0.002 /
    "---- Electromagnet force = kForce * I^2 / G^2"
    Constant  kForce = 44.8E-6
    Constant  Fmin = 1.0, Fmax = 1000.0
    Constant  Mass = 15.0

    "---- Current Amplifier characteristics"
    Constant  kIerr = 100.0    $ "Closed-loop gain"
    Constant  Vmax = 30.0      $ "Supply voltage"
    Constant  Imin = 0.0,  Imax = 20.0
    Constant  DeltaI = 0.005    $ "DAC resolution"

    "---- Disturbance characteristics"
    Constant  Ride = 1.5E-3, Tstart = 0.1, Ts = 0.0
    Constant  StpSiz = 1.0E-3, StpWid = 2.5
    Constant  MinGap = 0.5E-3, MaxGap = 10.0E-3

    "---- Feedback sensor time constants and quantization"
    Constant  TgapS = 0.80E-3, deltaG = 5.0E-6
    Constant  TaccS = 0.45E-3, deltaA = 5.0E-3

    "---- Acceleration signal high-pass filter (H/W) time constant"
    Constant  TaccF = 1.5

    "---- Absolute position controller feedback gains"
    Constant  Steady =      1.0  $ "-- s"
    Constant  Stiff  = 250000.0  $ "-- N/m"
    Constant  Damp   =   6500.0  $ "-- N/(m/s)"
    Constant  Vmass  =     15.0  $ "-- kg (virtual mass)"

    "---- Other time constants"
    Constant  Tfollo = 0.04  $ "Guideway following cutoff freq: 4 Hz"
    Constant  Tinteg = 1.6   $ "State integration cutoff freq: 0.1 Hz"
    Constant  Tforce = 0.0015 $ "Required force actuation time constant"

    "---- Initialise feedback signals"
    Idem = 0.0
    Gap = MinGap
    Gapmm = Gap * 1000.0
    Pi = 3.1415926
```

```
    "---- Simulation control"
    Constant  Tstop = 8.0
    Tdisp = Tstop - 2.0 * StpWid

End $ " of Initial"


Dynamic
    Termt(t .ge. Tstop)
    Td = t - Ts
    Tdisp = Tstop - 2.0 * StpWid

    Cinterval Tcomm = 10.0E-3  $  "Communication interval"
    Algorithm Ialg=3  $ "Euler integration"


    Derivative  Magnet  $  "Electromagnet and Current Amplifier"

      Maxterval Tmagss = 0.1E-3  $ "Integration step size"
      Nsteps Msteps = 1

      Procedural

        "---- Closed loop magnet current controller"
        Vmag = Bound( -Vmax, Vmax, kIerr * (Idem - Imag) )

        "---- Apply coil flux lag to electromagnet "
        Imag = RealPl( Lcoil(Gap)/Rcoil, Vmag/(Rcoil*Gap) ) * Gap

        "---- Apply eddy current flux lag to electromagnet"
        Flux = RealPl( Teddy(Gap), Imag/Gap )

        "---- Limits for magnet movement (clamped initially)"
        LowLim = FcnSw( t-Tstart, ride, MinGap, MinGap)
        UprLim = FcnSw( t-Tstart, ride, MaxGap, MaxGap)

        "---- Static electromagnet force characteristic"
        Fmag = Bound(Fmin, Fmax, kForce * (Flux*Flux) )

        "---- Magnet acceleration, velocity & position"
        Acc = 9.81 - (Fmag / Mass)
        DblInt(Pos, Vel =  0.0, Acc, 0.0, LowLim, UprLim)
        Gap = Bound( MinGap, MaxGap, Pos )
        Gapmm = Gap * 1000.0

        "---- Air gap and acceleration feedback sensors"
        GapFB = RealPl( TgapS, Gap, MinGap )
        AccFB2 = RealPl( TaccS, Acc )
        AccFB1 = RealPl( TaccS, AccFB2 )

        "---- Acceleration high-pass filter (analogue)"
        AccFB = AccFB1 - RealPl( TaccF, AccFB1 )

      End $ " of Procedural"
    End $ " of Derivative Electromagnet & Current Amplifier Model"
```

```
    Discrete  Contro  $  "Discrete time digital controller"

      Interval  Tsampl = 0.8E-3  $  "Controller sampling interval"
      Procedural

        "---- Position step input reference"
        GapRef = Ride + Bound(0.0, StpSiz, -1.0E6 * SIN(Pi*t/StpWid) )

        "---- Feedback sensor Quantization"
        AccQ = Bound( -9.81, +9.81, Qntzr(deltaA, AccFB) )
        GapQ = Bound( 0.0, 10.0E-3, Qntzr(deltaG, GapFB) )

        "---- Calculate Magnet acc, vel, pos & Track pos"
        AccHP  = AccQ - RealPl( Tinteg, AccQ )
        VelHP = RealPl( Tinteg, AccHP * Tinteg )
        PosHP = RealPl( Tinteg, VelHP * Tinteg )
        TrkHP = (GapRef + PosHP) - GapQ

        "---- Apply Guideway following filter"
        TrkF1  = RealPl( Tfollo, TrkHP )
        PosRef = RealPl( Tfollo, TrkF1 )

        "---- Absolute position controller"
        PosErr = PosHP - PosRef
        AccDem = Vmass * AccHP
        VelDem = Damp * VelHP
        PosDem = Stiff * PosErr
        SseDem = (Stiff/Steady) * LimInt(PosErr,0.0,-0.02,+0.02)

        "---- Steady-state, position, velocity, acceleration demand"
        Fdem = (9.81 * Mass) + AccDem + VelDem + PosDem + SseDem

        "---- Determine required magnet current"
        Fsqrt = SQRT( Bound( Fmin, Fmax, Fdem ) / kForce )
        Idem = Qntzr( deltaI, LedLag(Teddy(0.001), Tforce, Fsqrt*gapQ))

      End $ " of Procedural"
    End $ " of Discrete Controller"
End $ " of Dynamic "
End $ " of Maglev Simulation "
```