Appendix B

Electromagnet force control

B.1 Open-loop electromagnet force/voltage transfer function

The air gap flux, Φ, of the electromagnet is approximately proportional to the coil current, I, divided by the air gap, C, (see Equation 2.27). This nonlinear flux behaviour can be linearised for small perturbations and represented in the frequency domain by the Laplace transfer function given by:

$$\Delta\Phi(s) = k_{\phi i}\Delta I(s) - k_{\phi c}\Delta C(s) \quad \textit{where} \quad k_{\phi i} = \frac{\partial\phi(i_o,c_o)}{\partial i}, \quad k_{\phi c} = -\frac{\partial\phi(i_o,c_o)}{\partial c} \qquad \textbf{B.1}$$

The same treatment applied to the electromagnet force, F, (see Equation 2.25), which is approximately proportional to the square power of the air gap flux, gives:

$$\Delta F(s) = k_{f\phi}\Delta\Phi(s) \quad \textit{where} \quad k_{f\phi} = \frac{\partial f(i_o,c_o)}{\partial\phi} \qquad \textbf{B.2}$$

The dynamic relationship between electromagnet coil voltage, V, coil current, and air gap flux, (see Equation 2.26), is given by:

$$\Delta V(s) = sN\Delta\Phi(s) + R_{coils}\Delta I(s) \qquad \textbf{B.3}$$

where N is the number of coil turns, and R_{coils} is the coil resistance. Finally, the acceleration of the suspended load due to the electromagnet force is given by:

$$s^2\Delta C(s) = -\frac{\Delta F(s)}{m} \qquad \textbf{B.4}$$

where m is the total load mass. Equation B.1 is now rearranged to form an expression for the current in terms of the flux and air gap, this gives:

$$\Delta I(s) = \frac{1}{k_{\phi i}}\Delta\Phi(s) + \frac{k_{\phi c}}{k_{\phi i}}\Delta C(s) \qquad \textbf{B.5}$$

This current expression is substituted into Equation B.3, giving:

$$\Delta V(s) = sN\Delta\Phi(s) + \frac{R_{coils}}{k_{\phi i}}\Delta\Phi(s) + \frac{R_{coils}k_{\phi c}}{k_{\phi i}}\Delta C(s) \qquad \textbf{B.6}$$

Equation B.2 is now rearranged to form an expression for flux in terms of force, this gives:

$$\Delta\Phi(s) = \frac{\Delta F(s)}{k_{f\phi}} \qquad \textbf{B.7}$$

This flux expression is substituted into Equation B.6, giving:

$$\Delta V(s) = \frac{\Delta F(s)}{k_{f\phi}}\left(sN + \frac{R_{coils}}{k_{\phi i}}\right) + \frac{k_{\phi c}R_{coils}}{k_{\phi i}}\Delta C(s) \qquad \textbf{B.8}$$

Equation B.4 is now rearranged to give an expression for air gap in terms of forces by:

$$\Delta C(s) = -\frac{\Delta F(s)}{ms^2} \qquad \textbf{B.9}$$

This air gap expression is substituted into Equation B.8, giving:

$$\Delta V(s) = \frac{\Delta F(s)}{k_{f\phi}}\left(sN + \frac{R_{coils}}{k_{\phi i}} - \frac{k_{f\phi}k_{\phi c}R_{coils}}{k_{\phi i}ms^2}\right) \qquad \textbf{B.10}$$

This can now be rearranged to give the electromagnet force/voltage transfer function:

$$\frac{\Delta F(s)}{\Delta V(s)} = \frac{k_{f\phi}k_{\phi i}}{R_{coils}}\frac{s^2}{\left(s^3Nk_{\phi i}/R_{coils} + s^2 - k_{f\phi}k_{\phi c}/m\right)} \qquad \textbf{B.11}$$

For convenience, it is noted that (see Equations 2.27 and 2.29):

$$k_{\phi i} = \frac{\partial\phi}{\partial i} = \frac{\partial}{\partial i}\left(\frac{L_{coils}\,i}{N}\right) = \frac{L_{coils}}{N},$$

$$\therefore\ L_{coils} = Nk_{\phi i}, \qquad T_{coil} = \frac{L_{coils}}{R} = \frac{Nk_{\phi i}}{R} \qquad \textbf{B.12}$$

B.2 Closed-loop current feedback transfer function

The transfer function for the electromagnet current feedback loop is now analysed. Figure B.1 shows the configuration of the current feedback controller, where k_{amp} is the current loop gain, V_{mag} is the applied electromagnet coil voltage, I_{mag} is the electromagnet current, and I_{coils} is the component of the electromagnet current which produces the suspension force. The open-loop electromagnet transfer functions $I_{coils}/V_{mag}(s)$ and $I_{mag}/V_{mag}(s)$ are derived in Appendix A, Section 3.

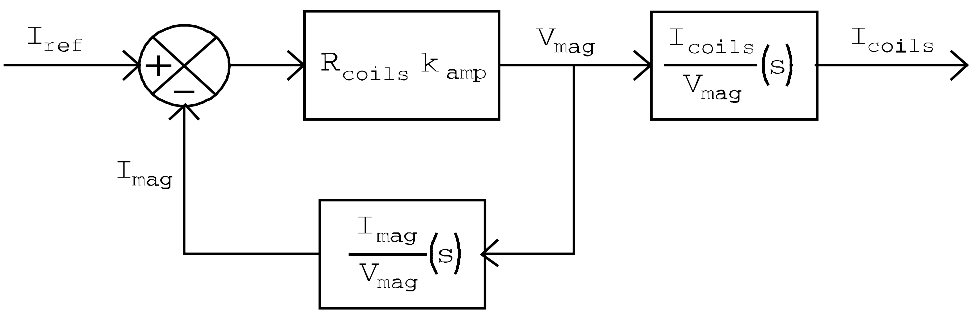

Figure B.1 Configuration of the closed-loop current feedback control

The closed-loop transfer function in terms of the Laplace operator *s*, is given by:

$$\frac{I_{coils}(s)}{I_{ref}(s)} = \frac{R_{coils}\,k_{amp}\,\frac{I_{coils}}{V_{mag}}(s)}{1 + R_{coils}\,k_{amp}\,\frac{I_{mag}}{V_{mag}}(s)} \qquad \textbf{B.13}$$

Expanding the two open-loop transfers functions gives:

$$\frac{I_{coils}(s)}{I_{ref}(s)} = \frac{R_{coils}\,k_{amp}\left(\frac{1}{R_{coils}}\,\frac{1}{1 + s\left(T_{coils} + T_{eddy}\right)}\right)}{1 + R_{coils}\,k_{amp}\left(\frac{1}{R_{coils}}\,\frac{1 + sT_{eddy}}{1 + s\left(T_{coils} + T_{eddy}\right)}\right)} \qquad \textbf{B.14}$$

This can be simplified to give:

$$\frac{I_{coils}(s)}{I_{ref}(s)} = \frac{k_{amp}}{1 + s\left(T_{coils} + T_{eddy}\right) + k_{amp}\left(1 + s\,T_{eddy}\right)} \qquad \textbf{B.15}$$

For large values of feedback gain k_{amp}, this can be approximated by:

$$\frac{I_{coils}(s)}{I_{ref}(s)} \approx \frac{1}{1 + s\left(\dfrac{T_{coils}}{k_{amp}} + T_{eddy}\right)} \qquad \textit{where} \quad k_{amp} \gg 1 \qquad \textbf{B.16}$$

Equation B.16 shows that the use of current feedback reduces the time constant due to the electromagnet coil, but not the time constant due to the eddy currents circulating within the cores.

B.3 Matlab models for root loci generation

```
%  MatLab Script  f_locus.m
%
%  This script produces a root locus plot for force feedback
%
%         Author:  N. S. McLagan
%  Last modified:  11th December, 1990
%

% Electromagnet/amplifier parameters
Tflux = 0.116
kc = 950000
mass = 15

% Gain factor range
kmax = 120
k = [ 0:0.005:0.135, 0.1409811442, 0.141, 1 ];

% H(s) numerator & denominator
num = [ kmax, 0, 0 ]
den = [ Tflux, 1, 0, -kc/mass ]

p = rlocus( num, den, [0] )
r = rlocus( num, den, k );
a = rlocus( num, den, [1] );
axis([-200, +100, -100, +100])
plot( real(r), imag(r), '-' )
text( real(p)-1.5, imag(p)-2.4, 'X')
text( real(a(2))-1.5, imag(a(2))-2.4, '>' )
text( real(a(3))-1.5, imag(a(3))-2.4, '<' )
grid

!del f_locus.met
meta f_locus.met            % Put current plot into temporary metafile
!gpp f_locus /dhpgl          % Invoke GPP to convert to HPGL
```

```
%  MatLab Script  c_locus.m
%
%  This script produces a root locus plot for air gap feedback
%
%         Author:  N. S. McLagan
%  Last modified:  12th December, 1990
%

% Electromagnet/amplifier parameters
Tflux = 0.0057
kc = 950000
mass = 15

% Gain factor range
kmin = 0
kmax = 2
k = [ kmin:0.05:0.9, 0.92800303825, 1.000001, 1.05:0.05:kmax ];

% H(s) numerator & denominator
num = [ 0 0 kc/mass ]
den = [ Tflux 1 0 -kc/mass ]

p = rlocus( num, den, [kmin] )
r = rlocus( num, den, k );
a = rlocus( num, den, [kmax] );
axis([-400, +200, -200, +200])
plot( real(r), imag(r), '-' )
text( real(p)-3, imag(p)-4.8, 'X')
text( real(a(1))-3, imag(a(1))-4.8, '<' )
grid

!del c_locus.met
meta c_locus.met              % Put current plot into temporary metafile
!gpp c_locus /dhpgl           % Invoke GPP to convert to HPGL
```